Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01

WMA13 Report June 2023

Overview

The paper gave good coverage of the specification for the IAL Pure Mathematics Core 3, with a range of items of varying difficulty. The most challenging areas were questions 4(c), 6(d), and question 10. Along with question 1, where a neglect to mentioned continuity continues to cost many students a mark, these were the only questions where the modal score was not full marks.

Students responses were generally well-presented and legible, though the ability to confidently manipulate algebraic expressions was lacking in some students work. There was little evidence that students were unable to complete the paper in the allotted 90 minutes, though some question 10's were poorly answered, possibly due to hurrying at the end of the paper.

Question 1

This question was well answered by the vast majority of students. Though the modal score was 4/5 (achieved by just over 40%), over one third achieved full marks, with the lack of reference to continuity in (a) being the primary culprit for the loss of a mark.

A very small number of students started off (a) or (b) by showing how the Iterative formula was derived, but this was not necessary. Students should make a careful read of questions to know what is being required.

In part (a), the vast majority were able to score at least the first mark, though, as noted, a smaller proportion managed to score both marks. The majority of cases scoring 1 out of 2 failed to state that the function was continuous but some failed mention that there was a root in the interval, while there were occasional miscalculations of g(3) and /or g(4). Students used the expected interval [3,4] for their test — although a narrower interval was acceptable no instances of this were reported. The method of the sign change is well known, and made clear in most cases, the periphery is where some students went awry.

In part (b), the vast majority showed the substitution, or had a correct value, and generally answered the question well and gaining all three marks. Only a very few did not understand how to find a root using an iterative formula, some losing all three marks due to failing to show any working and having incorrect values. The wrong number of decimal places was the most common reason for loss of the final mark, though there was a small number of students who gave the correct value of α without first finding x_1. These were given benefit of the doubt and awarded M1A0A1 even though it is unclear if they used repeated iteration or used a calculator to find the root of the given equation, but students should be warned to make working clear as benefit of the doubt may not always be given in such cases.

Question 2

Most students answered this question well with nearly 50% scoring the modal full marks. Aside from this score the spread of marks was fairly uniform with some making no progress, but most able to score in excess of 3 marks.

In part (a)(i), writing down the required straight-line equation from the axis intercepts shown on the given log-log graph proved to be straight forward and was generally done correctly for one mark. If this was not achieved, for example, a few students simply wrote $y = 4 - 2x$, then there was usually very little progress made. Use of log notation was generally good, with the base number shown, but the argument was often written as a superscript within the log.

In part (a)(ii), students had to find the exact value of the dependent variable (T) given a value for the dependent variable ($x = 216$). The majority of students were able to achieve first M mark by substituting $x = 216$ to an appropriate equation linking T and x and most went to to proceed to a value of T, though the log work was not always correct. As the value given was a power of the log function base number (6) and many students simplified fully in one step to reach $\log_6 T = -2$ and from here usually deduced the correct answer. Incorrect work was often to do with not knowing how to deal with a base 6 number.

In part (b), students were required to find an equation linking T and x. Many made T the subject via correct use of log rules but this was not required, with $Tx^2 = 1296$ being an acceptable form. Almost all students gained the first mark for making a first step towards the answer via a correctly applied log rule, which was usually the index rule. Many students made heavy work of the solution by raising the whole of the right-hand side to base 6 before simplifying, indicating a lack of strategic thinking in their approach. Nonetheless many students did reach the answer by combining logs first. The dM mark was where things often went wrong, with the log rules not well understood by many students. If the method marks were achieved, then the answer was usually correct, students scored all three marks.

Another, though less common, error throughout the question was being confused by log base 6, with some attempts to use log base 10 or 'e' and 'ln'.

Question 3

All parts of this question were attempted by most students. The modal mark of full marks was achieved by about 30%, and a further 20% dropping just one mark, so not as well answer as the previous questions but still proved a good source of marks overall.

In part (i), students were required to differentiate $\ln\left(\sin^2 3x\right)$ and the vast majority of attempts successfully applied the reciprocal rule for differentiating $\ln x$. There was somewhat less success with the chain rule aspect of the solution, though. Of those who did obtain the correct derivative, some did not fully simplify the result and it was not uncommon to see common factors left in the numerator and denominator which lost the final mark in this part. Although not required for full marks, many stopped short of expressing the answer in terms of $\cot\left(3x\right)$, preferring to leave it in terms of sine and cosine functions. Though the main scheme approach was the preferred method, the different methods shown on the mark scheme, with trigonometric identities or log power law used first, were all commonly seen, each with varying degrees of success.

For part (ii)(a), most were able to apply the chain rule successfully initially, but there were often errors in simplification of the differentiated expression, for example $6\left(3x^2-4\right)^5\times 6x$ was frequently simplified to $12x\left(3x^2-4\right)^5$, sometimes missing the x before or after simplification, and so marks were often achieved at the unsimplified stage (if correct) which was sufficient here. There were only occasional instances of students integrating rather than differentiating.

In part (ii)(b), students were required to integrate an expression that was effectively the result of the previous differentiation, and so to use recognition. Hence students did not need to be able to integrate using the chain rule, lightening the demand of the question. Some students elected to restart the question and integrate using a substitution (or by recognition of a chain rule derivative), and most attempts at this were successful, although it was not uncommon in such cases for an extra x term to appear in the result of the integration which ruled out further marks. A few students attempted to integrate by parts, making little progress with the question. There were a number of slips and errors observed including omitting the power of 6 when applying limits and the assumption that the '0' limit would yield a zero contribution to the definite integral. Students would be well-advised to demonstrate the substitution of the limits of their integral as this can ensure that credit can be given even if slips are made when calculating the end result. A small number of students wrote the answer only, with no working, likely having used their calculators to obtain the correct value and this scored no marks. Again, it should be stressed that method must be shown when answering questions in order to guarantee full marks. Very few were thrown by the fact that the required definite integral resulted in a negative number, though some did make the answer positive, thinking of it as an area. These were given credit as long as the correct negative value was first seen.

Question 4

This was an accessible question with almost all students making some attempt, but the modal mark was only 4/6, achieved by 25%, with part (c) causing the main issues. Only about 20%

scored full marks, though this was the second most common score, with other scores being evenly spread aside 5/6, which was rare.

Part (a) was answered correct by the majority due to the leniency of the scheme allowing *y*, f, f(x) etc., although a few lost this mark because they used > rather …. For a topic that often causes many problems, it was good to see many obtain the correct range here. Correct notation was used, with a very few giving the range as an inequality in *x*. Where students drew a graph to support their findings this almost always resulted in the correct range being found.

Part (b) proved to be more challenging, although it was still completed successfully by the majority of students. These students appreciated that inverse functions are reflective in the line $y = x$ and used this to at least draw a graph in the 1st and 2nd quadrant gaining the method mark. Of those who got the first mark for the position, many lost the accuracy mark as their graph did not have the correct curvature. Occasionally the curvature had an increasing gradient or tended back towards the *x*-axis resulting in A0, while a few drew the reflection of the given graph in the y-axis to give a complete quadratic curve. Some reflected in the *x*-axis and a surprising proportion made no attempt at the sketch.

Part (c) proved to be a good discriminator. The students that were successful here realised that the point of intersection was on the line $y = x$ and set $\mathrm{f}(x) = x$ to obtain a quadratic equation which they solved with ease, giving an exact answer and rejecting the negative solution. The majority however, attempted to find $\mathrm{f}^{-1}(x)$, usually correctly, then put $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ and ended up with a quartic. These were able to score the independent B mark provided they had set up a correct equation but were not able to find an exact answer to their quartic equation without significant extra work, only very rarely seen, to factorise it, so could not access the method nor the accuracy marks. Reliance on calculators to find approximate solutions to equations is very common, but not acceptable where exact answers are required, and so students need to ensure they have a suitable strategy to find exact answers when they are asked for.

<u>Question 5</u>

Another very accessible question, being ranked third easiest question on the paper. The modal score was full marks and scored by nearly 40% of students, with more than another 15% scoring 6/7. There were a few who either did not attempt the question at all, or made no significant progress, with 10% scoring no marks.

Part (i) proved to be the more difficult of the two parts to score marks. A significant number started by multiplying out the brackets and could make no further progress. Ignoring the bracket $(x - 2)$ and so losing the solution $x = 2$ was also common. The majority were able to solve $\sqrt{3}\sec x + 2 = 0$ by converting to $\cos x$ and showing sufficient working to obtain the solution $\frac{5\pi}{6}$, though there were some errors in rearranging first, such as leading to solving $\cos x = \frac{\sqrt{3}}{2}$

instead of $-\frac{\sqrt{3}}{2}$. Occasionally $\frac{\pi}{6}$ or another value was also given as a solution, so losing the last mark. A few gave their answer in degrees, also losing the A mark, but this was not common. Heed should be paid to the range the answers are required to be in to know what mode to be solving in.

Part (ii) was better attempted on the whole with most students able to earn at least the method marks here for using the relevant double angle identity to achieve a quadratic and then solve it. Most students replaced $\cos 2\theta$ using a correct double angle identity although a few made sign errors. These students were still able to reach a quadratic equation which they then solved to find values for θ. A small number used $1-\sin^2\theta$ for the double angle, but still achieved a quadratic equation which they then solved. There were a few instances where students correctly found the acute angle but either used $360 - \theta$ or $180 + \theta$ to find the second value. A few who had the correct quadratic lost the final accuracy mark for rounding errors but most produced accurate work and earned all of the marks. A number of students also found additional solutions in the range which cost them the final accuracy mark, the most common being 195°. Some offered their answers in radians, again the mode of the calculator should be checked at the start of the question.

Question 6

Though overall this question proved accessible, and a good source of marks, full marks was actually relatively rare, achieved only by about 15%. The modal score of 6/9 was awarded about 20% of the time, with scores of 5, 6 and 8 all also common (>12% each), but by contrast less than 20% scored fewer than 5 marks and only about 2.5% scored no marks at all. So there was certainly good access to the most marks, but part (d) in particular was a good discriminator on the paper.

Part (a) was very well done with the majority giving both coordinates correctly. Very few cases of incorrect coordinates were seen, and where they were it was usually the y value.

In part (b), most showed a good understanding of the correct process of composite functions. The most common error was in not dealing with the modulus signs correctly rather than being unable to process a composite function, and many provided sufficient working and scored M1A0 here in such cases. Some opted to work out a complete formula for ff(x) first then substituted $x = 0$. These attempts often led to errors and resulted in an incorrect value for ff(0), also scoring M1A0. However the majority did obtain the correct answer for this part.

In part (c), most attempted to solve a correct equation and, although there were occasional slips in the algebra, scored the first mark. However, some changed the sign only in the first term under the modulus and such an error meant the method mark was not scored, as they were not solving a correct equation. The common problem in this part was that many who were solving the correct equation also solved $3(x-2)-10<5x+10$ leading to $x > -13$ and while some rejected this solution giving only $x>-\frac{7}{4}$, others left both inequalities or formed an inequality from both sets of values, losing the accuracy mark. Some students also rejected the wrong value. The given sketch could have helped them to see what solutions were valid but there was little evidence that this was used. Some had an incorrect inequality sign or left it as an equality.

Part (d) was the most challenging part and very few students understood the graphical significance of $\mathrm{f}(|x|)$, so fully correct answers were rare. The solution $x=\frac{16}{3}$ was often found, but few seemed to know that for $\mathrm{f}(|x|)$, the negative x part of the graph is a reflection of the positive x part of the graph, so $x=-\frac{16}{3}$ is also a solution. Most instead attempt to solve an equation in x or $|x|$ with varying degrees of success in setting up a correct equation for the second solution. Those who reached $|x|=\frac{16}{3}$ usually then found both values for x. But many had equations leading to $x=\pm\frac{4}{3}$. Some had all 4 of these values as their answer. Those that did a sketch of $\mathrm{f}(|x|)$ tended to spot the reflective properties of the graph and its intercepts.

Question 7

This question was attempted by the vast majority of students most of whom gained at least some marks, and the modal mark of full marks being scored by over 40% of students meant that this was one of the most accessible questions of the paper. Another 25% were able to score 6 or 7 marks for the question, with fewer than 10% scoring less than 3 marks (half of whom scored no marks, usually by a non-attempt).

Part (a) was generally well answered and almost all earned the first mark for correctly stating $A = 2500$. Most were able to set up and solve a correct equation to obtain a value for k. A small proportion of students made arithmetical mistakes, but most substituted in the values and rearranged correctly and full marks were often awarded. Occasionally, students missed that k was required to four significant figures and 0.173 was stated rather than 0.1733. This was often preceded by the correct exact value for k thus avoiding the loss of a mark.

Part (b) provided a more varied standard of responses between students. A significant minority of students were unsure about what was required here. Some, perhaps missing, or misunderstanding, the reference to ‘rate of decrease’ substituted $t = 5$ directly into N. This was sometimes followed by an attempt at a percentage change calculation, other times by an attempt to treat N as a linear function by calculating the difference between N at $t = 0$ and N at $t = 5$ followed by dividing by 5. Both approaches earned no marks. Those students who recognised differentiation was required were far more successful and the fairly simple differentiation was achieved by most although occasionally the application of the chain rule was omitted by some which led to an incorrect coefficient of the exponential term and a corresponding incorrect value for the rate of decrease of the bacteria. Some students stated a value of −1790 rather than 1790 but this was condoned as was giving the value to more than three significant figures.

In part (c) students were asked to find the time at which the two population sizes were equal. Most students were able to earn the first mark for setting up the equation by equating the two expressions for N. However, the demand and challenge presented by the log work required to solve the equation meant that many were unable to earn full marks here, the usual 2 marks lost in those scoring 6/8. The safest approach was to rearrange the equation and apply the rules of indices in order to obtain an equation with a single exponential term before applying natural logarithms. Usually far less successful approaches involved taking logarithms of both sides as a first step which often led to incorrect log work and loss of marks. Premature rounding also caused some issues here with answers of 4.12 being occasionally given.

Question 8

Although the modal mark, scored by just over 25% of students, was full marks on this question, it did provide much more of a degree of challenge than many of the preceding question. The next most common scores (scored by10-15% each) were 2, 8 or 0 out of 9, with the other scores less common. The scores of zero were more often attempts at (a) that faltered and before giving up on the question, rather than non-responses - the students not picking up that part (b) could be deduced from the given form of answer in part (a).

In part (a), the first two marks for an unsimplified answer to the differentiation were generally achieved. The accurate factorisation of the unsimplified form was usually done well, not least because the required form of the answer was given in the question. A few made little or no attempt to factorise, and some erroneous results were seen. Other values for A seen included 4 and 6. A much smaller proportion obtained the result through expanding the brackets, $(2x + 1)^2$ and $(2x + 1)^3$, and factorising the resulting cubic polynomial to yield the required expression, creating needless extra work, and often resulting in calculator use to solve the cubic to be able to factorise.

In part (b), students generally realised what to do and answered this question well. Most were well schooled in finding stationary points and solved their equation to find the 2 values of x and substituted them into $f(x)$ to find the coordinates of the maximum and minimum. A few cost themselves the accuracy mark by writing $\frac{27}{8e}$ as a decimal without exact value shown, but most used the correct exact answer. Common errors observed in this part were sign errors, with $x = \frac{1}{2}$, for example, being a common value for x used. Another error seen was a maximum y

coordinate stated as $\frac{27}{8}e^{-4x}$, the x in the exponent not being substituted. On rare occasions, students tried to solve their equation f'(x) = 0 by multiplying out all brackets but this was usually unsuccessful in achieving a correct suitable value for x to use. Some did not attempt a y value at all in part (b), but were allowed credit for the method if they found one in part (c), though often only one was found, so the final A could not be awarded as the second correct coordinate was not seen.

In part (c), there were many scripts seen where there was no attempt at this part but those that did attempt it generally got at least the first follow through the mark. Both marks were follow-through, so that erroneous results from (b) could still accrue marks in (c), though the second of the marks did require the correct maximum point to have been selected (following through only on their y ordinate at $x = \frac{1}{4}$).

Many knew that for the new function g(x) = 8f(x – 2), they could quickly state the maximum point by adding 2 to the x coordinate and multiplying the y coordinate by 8. However, some did not know or work out which was the maximum point and did this for both points, so losing the A mark. Some subtracted 2 instead of adding in the first ordinate, but dealing with the y ordinate was usually successful. In rare occasions the coordinates were also swapped, forfeiting both marks. Where a non-exact y coordinate was given in (b), that value, multiplied by 8, was accepted here.
A small number of students started again trying to differentiate the new function g(x) and finding its stationary points. This was time consuming and, although it was possible to arrive at the correct stationary point, most quickly abandoned this approach.

Question 9

This proved to be another highly accessible question, with full marks as the modal score (by over 25%) comfortably outperforming other scores, with 3, 9, 8, 0 or 1 mark, all scored by about 10% being other common scores.

In part (a), the majority of students scored full marks using a variety of strategies of varying efficiency - some very efficient approaches per the main scheme method. To see a trigonometric proof done well so often was pleasing. Some students did fail to show sufficient work at the end of the proof, missing the required $\frac{1}{\sin x}$ before the final answer. All methods on the scheme were regularly seen, with no method being particularly prone to error. Unsuccessful attempts were usually able to score the first method mark for either combining the fractions successfully or applying one correct double angle formula, meaning zero scores here were generally for an omission of this part.

In part (b), the majority of students successfully formed an equation in cot x and were able to score at least the method marks, and usually the first 4 marks if not full marks. A few students instead tried to form an equation in either cosec θ or sin θ but these were rarely successful and made little progress. It was relatively common to not give $\frac{\pi}{4}$ as an exact answer, showing only the approximation 0.785, which was penalised the final accuracy mark. Students should note that if "where appropriate" is mentioned it means some of the solutions can and should be given exactly. Some errors noted in this part included that students saw the connection between parts but replaced the left hand side with just cosec θ, not realising that it had to be squared, while others had $\left(\text{cosec}^2 x\right)^2$ getting confused with what was being squared.

Though part (c) was the least successfully answered part overall in this question, it was nevertheless well attempted by many students, showing an understanding to again use part (a) to replace the integrand and use the reverse differentiation of cosec x. Most students achieved the integral directly from the known result in the formula book and achieved both marks. However, some students did not realise the result could be found from their Formula Booklet, and instead used a substitution and changed the limits, which were not always correct, and if the substitution was not undone lost the final mark. The substitution approach often gave the integral as $-\frac{1}{\sin x}$, which is of course also correct and led to successful solutions. However, there were also numerous students who did not show any form for the integral and simply used their calculators to evaluate directly scoring M0A0. Others gave the answer in decimals - possibly from a calculator if they were not confident in their integration. If the question says an exact answer is wanted then that is what is required and so must be given.

Question 10

Despite a few blank responses, possibly indicating a lack of time, most students attempted this question and obtained at least some marks. However, and not surprisingly as the last question, it was one of the least well answered questions on the paper with nearly 25% scoring the model mark of 3/8, while a little over 15% scored 7/8 and similar scored 0/8. The spread across other marks was uniform. Part (a) was usually well approached, but many did not realise what was needed in (b).

In part (a), the majority of students recognised that they could use the quotient rule, and many did so successfully here despite x being in terms of y. Few incorrect 'quotient rules' were seen but it is always advisable for students to quote the rule they are using before applying it to a particular function in case of slips in substitution. Some students did attempt to carry out an algebraic division prior to differentiating and often did so accurately but in some cases these students struggled to differentiate the resulting form successfully. Use of the product rule was rarely seen.

Complete simplification of the differential proved to be something of a challenge for many. Most did manage to multiply out the brackets on the numerator, gather terms and re-factorise but rarely was a full simplification seen and arithmetical errors were not uncommon. Some students lost a factor of ‘2’ in the numerator while others believed that the factorised brackets should cancel and so manipulated the factor of $(y-1)$ to $(y+1)$ or vice versa; thus losing the quadratic numerator to a linear form. Ultimately, even students who obtained a correct form of $\frac{6(y^2-2y-3)}{(3y^2-3)}$ did not often realise that a factor of 3 could be cancelled from numerator and denominator.

In part (b), students were asked to find the equations of two tangents to the curve parallel to the y-axis. Most students realised that they needed to set $\frac{\mathrm{d}x}{\mathrm{d}y}=0$ and most realised that it was sufficient to set the numerator equal to 0. Some with simplifying errors in part (a) were unable to make progress here if they had obtained a linear numerator in part (a). Many, however, were able to solve their resulting quadratic to find two values for y. A minority of students believed that they should be working with $\frac{\mathrm{d}y}{\mathrm{d}x}$ here rather than $\frac{\mathrm{d}x}{\mathrm{d}y}$ which of course, if treated correctly, could still lead to a correct answer but these students often failed to do so.

Those students with the correct quadratic usually solved it correctly for the 2 correct values of y, substituted correctly to find 2 values of x, but did not always go on to give the correct answer. It often appeared that, at this stage, students were not returning to the body of the question in order to remind themselves of the demand of the question. It was not uncommon to see an answer of two coordinate pairs $\left(-\frac{4}{3},3\right)$ and $(4,-1)$ stated rather than the equations of the tangents $x=-\frac{4}{3}$ and $x = 4$. It was clear that some students did not believe $x=-\frac{4}{3}$ and $x = 4$ to be the desired equations and such students went on to try to find the equations of non-vertical ‘tangents’ to the curve trying to use $y - y_1 = m(x - x_1)$, while a small number giving lines parallel to the x-axis.